Calculation of the minimum number of sheets for the determination of paper reflectivity (R_∞)

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〔Abstract〕

The minimum number (n) of sheets for the determining of paper reflectivity (R_∞) can be calculated by the use of equation $Ti^n (1-R)^{n+1} \longrightarrow 0$ exactly. **Bulletin Exp. For. NCHU No. 89-91 (March, 1986)**

〔Key words〕

Reflectivity, Opacity, Reflectance, Transmittance.

測定紙張反射性(R_∞)所需最少紙張數目之計算

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〔摘　要〕

測定紙張反射性（R_∞）時所需最少紙張數目(n)可藉著$Ti^n (1-R)^{n+1} \rightarrow 0$公式正確地求出。中興大學實驗林研究報告第七號 89~91（民國75年3月）。

〔關鍵字〕 反射性，不透明度，反射率，穿透率。

1. Introduction:

Opacity with a paper backing (printing opacity) is the most common opacity measurement; it measures the property that is visually judged as " show-through " when a number of sheets printed on one side are stacked in a pile. It is a ratio of the reflectance of a single sheet of paper backed by a black cavity to the reflectance of a think pad of the same paper. The ratio is expressed as Ro/R_∞.

One objection to the use of printing opacity as a test has been the need of a thick pad of the paper for measuring R_∞. The reflectivity of paper is the reflectance of layer so thick that further increase in thickness does not change the reflectance. The minimum number of low-basis weight sheets for the determination of paper reflectivity can not be predicted precisely unless we try patiently.

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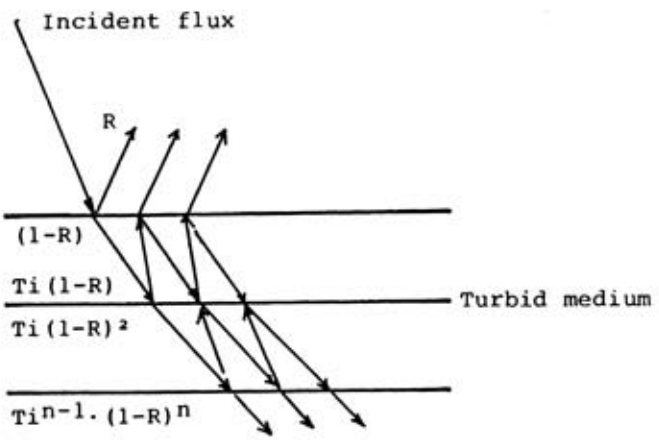
A laminar layer of paper reflects a fraction, R, of the incident flux from its front surface, the remainder, (1-R), penetrates the surface, and a fraction Ti (1-R), reaches the opposite face, where Ti is the internal transmittance of the layer. Various fractions of this are multiply reflected between the top and the bottom of the layer as shown in Fig. 1.

Theoretically, if no further change in reflectance of the top layer of a pile of sheets, then the transmittance of n'th layer of sheet is close to zero, i.e, $Ti^n\ (1-R)^{n+1} \longrightarrow 0$ (1)



Plane-parallel faces.

Fig. 1. Light goes through turbid medium. (Ref. 1)

圖 1　光行經濁介質之途徑

2. Equations

When the value of $Ti^n\ (1-R)^{n+1}$ is less than 10^{-6} , Find n= ?, n = 1, 2, 3, 4.....n.

$$\text{Where } Ti = \frac{b}{a \sinh bsW + b \cosh bsW} \text{ (Ref. 2)}$$

$$sW = \frac{1}{1/R_\infty - R_\infty} \ln \frac{1-Ro \cdot R_\infty}{1-Ro/R_\infty}$$

$$a = \frac{(1/R_\infty + R_\infty)}{2} \qquad b = \frac{(1/R_\infty - R_\infty)}{2}$$

$$R_\infty = a - (a^2-1)^{\frac{1}{2}}, \quad a = \tfrac{1}{2}\left[R_{0.89} + (R_o - R_{0.89}+0.89)/(Ro \cdot 0.89)\right]$$

3. Sample problem

Calculate the mininum number of blue paper (Basis weight 26 g/m²) for the determination of R_∞ with Ro = 0.46, Ro.89 = 0.78.

(Solution)

Entering the given values into the equation $Ti^n (1-R_{0.89})^{n+1} < 10^{-6}$, n = 6.

The observed values of $R_{0.89}$ for n'th layer of paper sheet are listed as follows :

Number of sheets n	$R_{0.89}$ Observed value
1	0.780
2	0.715
3	0.688
4	0.675
5	0.668
6	0.662
7	0.662

λ = 572 nm.

4. References

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